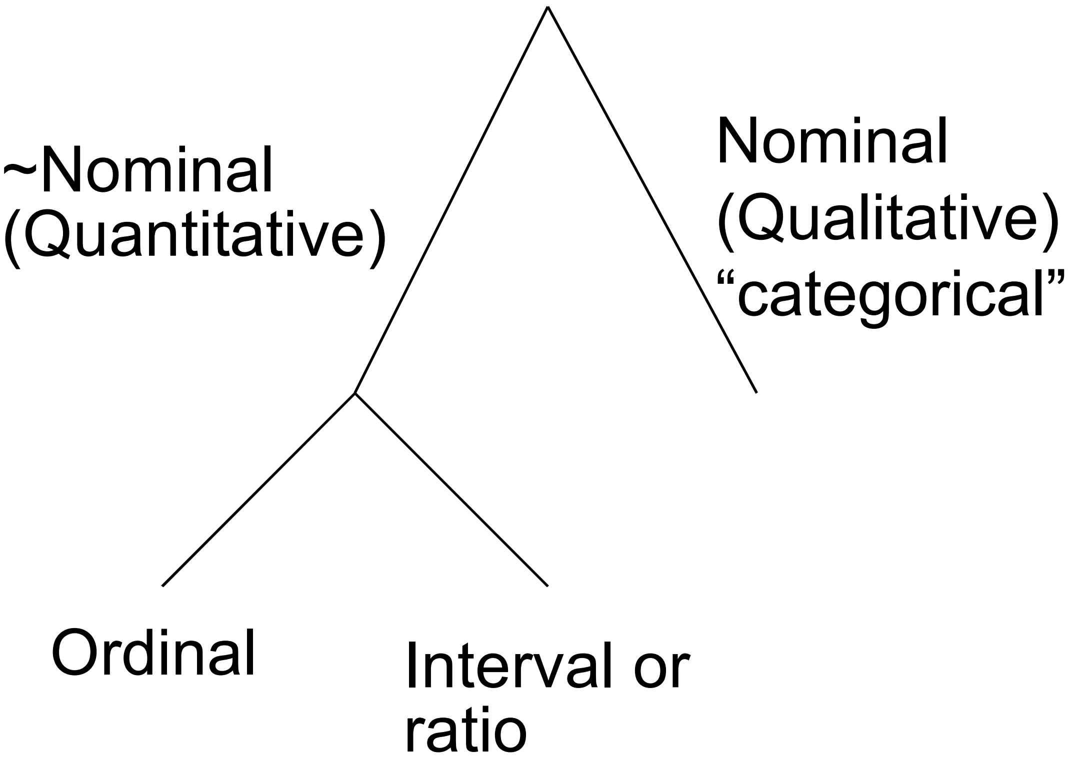
Introduction to Descriptive Statistics

17.871

Types of Variables



Describing data

	Moment	Non-mean based measure
Center	Mean	Mode, median
Spread	Variance (standard deviation)	Range, Interquartile range
Skew	Skewness	--
Peaked	Kurtosis	--

Population vs. Sample Notation

Population	*Vs*	Sample
Greeks		Romans
μ, σ, β		s, b

Mean

$$\frac{\sum_{i=1}^{n} x_i}{n} \equiv \mu \equiv \overline{X}$$

Variance, Standard Deviation

$$\sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n} \equiv \sigma^2,$$

$$\sqrt{\sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n}} \equiv \sigma$$

Variance, S.D. of a Sample

$$\sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n-1} \equiv s^2,$$

Degrees of freedom

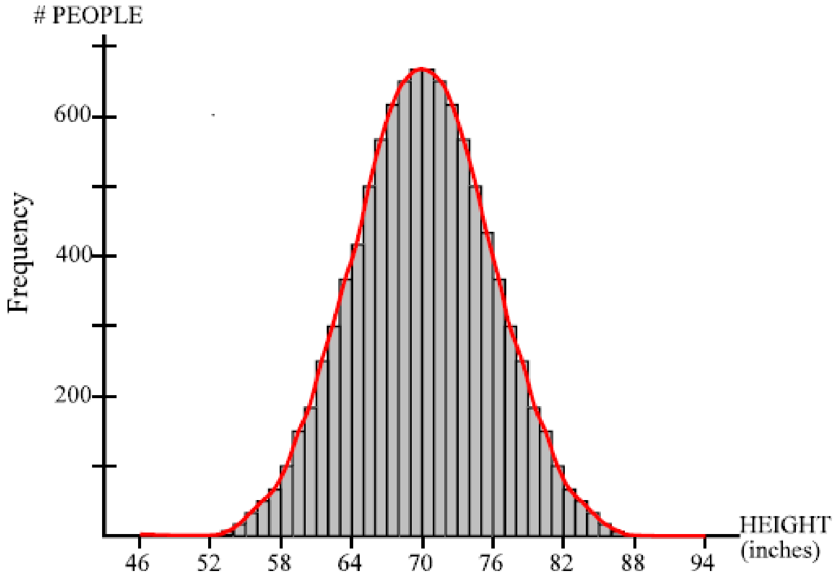
$$\sqrt{\sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n-1}} \equiv s$$

Binary data

$$\bar{X} = prob(X) = 1 = \text{proportion of time } x = 1$$

$$s_x^2 = \bar{x}(1-\bar{x}) \Rightarrow s_x = \sqrt{\bar{x}(1-\bar{x})}$$

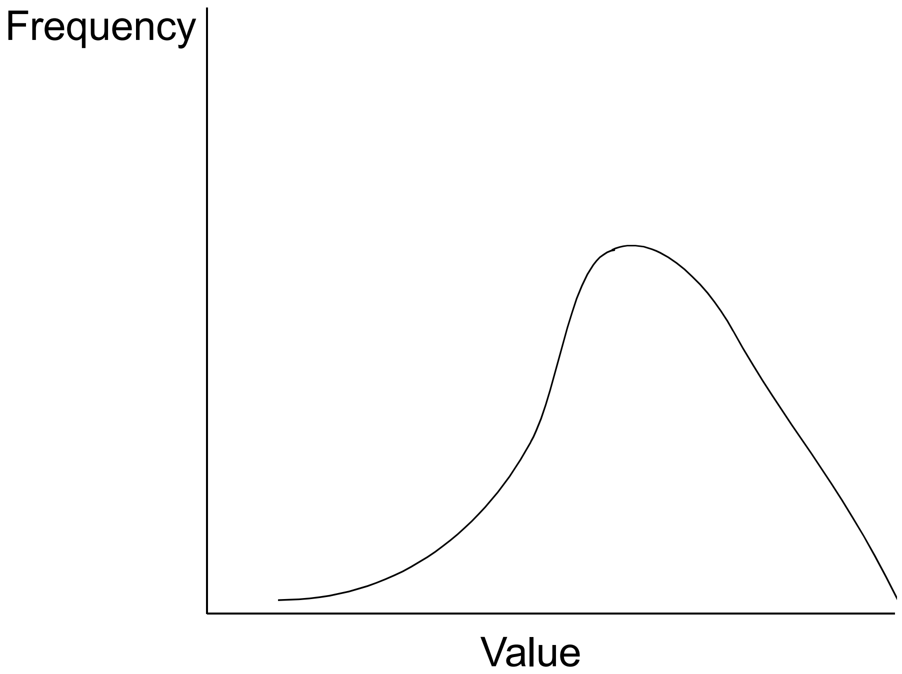
Normal distribution example



- IQ
- SAT
- Height

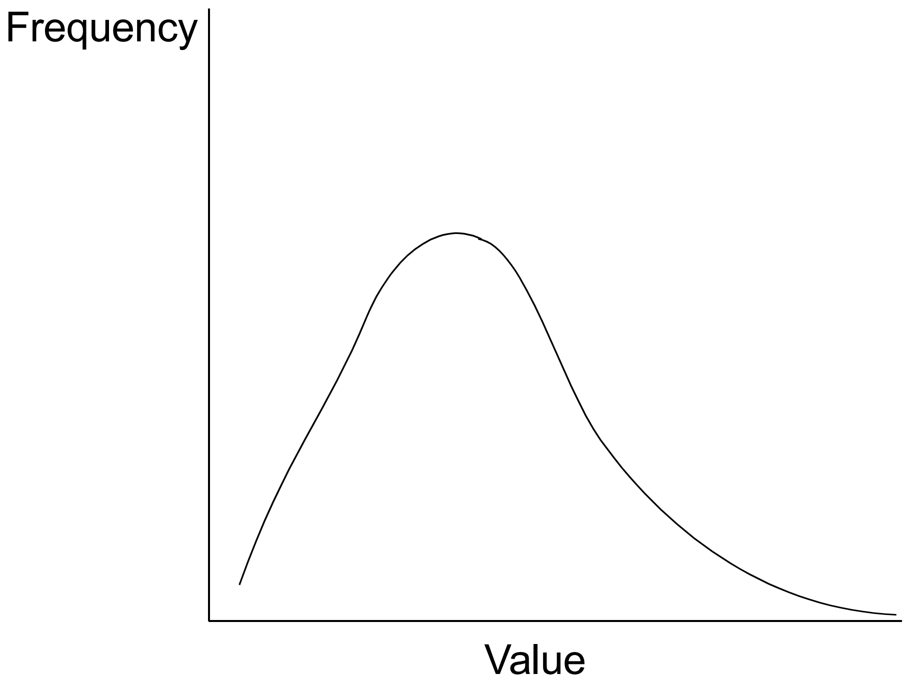
- "No skew"
- "Zero skew"
- Symmetrical
- Mean = median = mode

Skewness
Asymmetrical distribution



- GPA of MIT students
- "Negative skew"
- "Left skew"

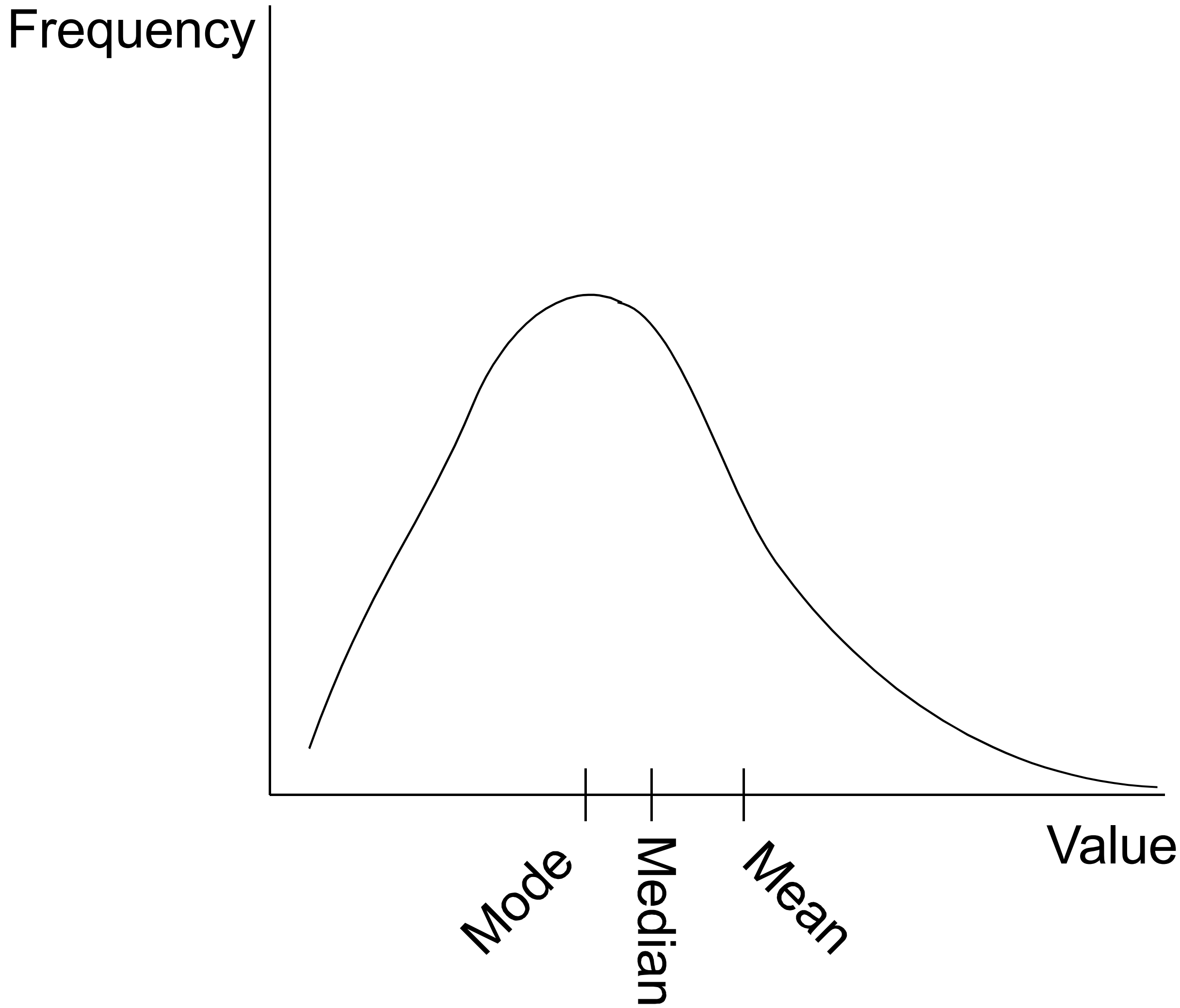
Skewness (Asymmetrical distribution)



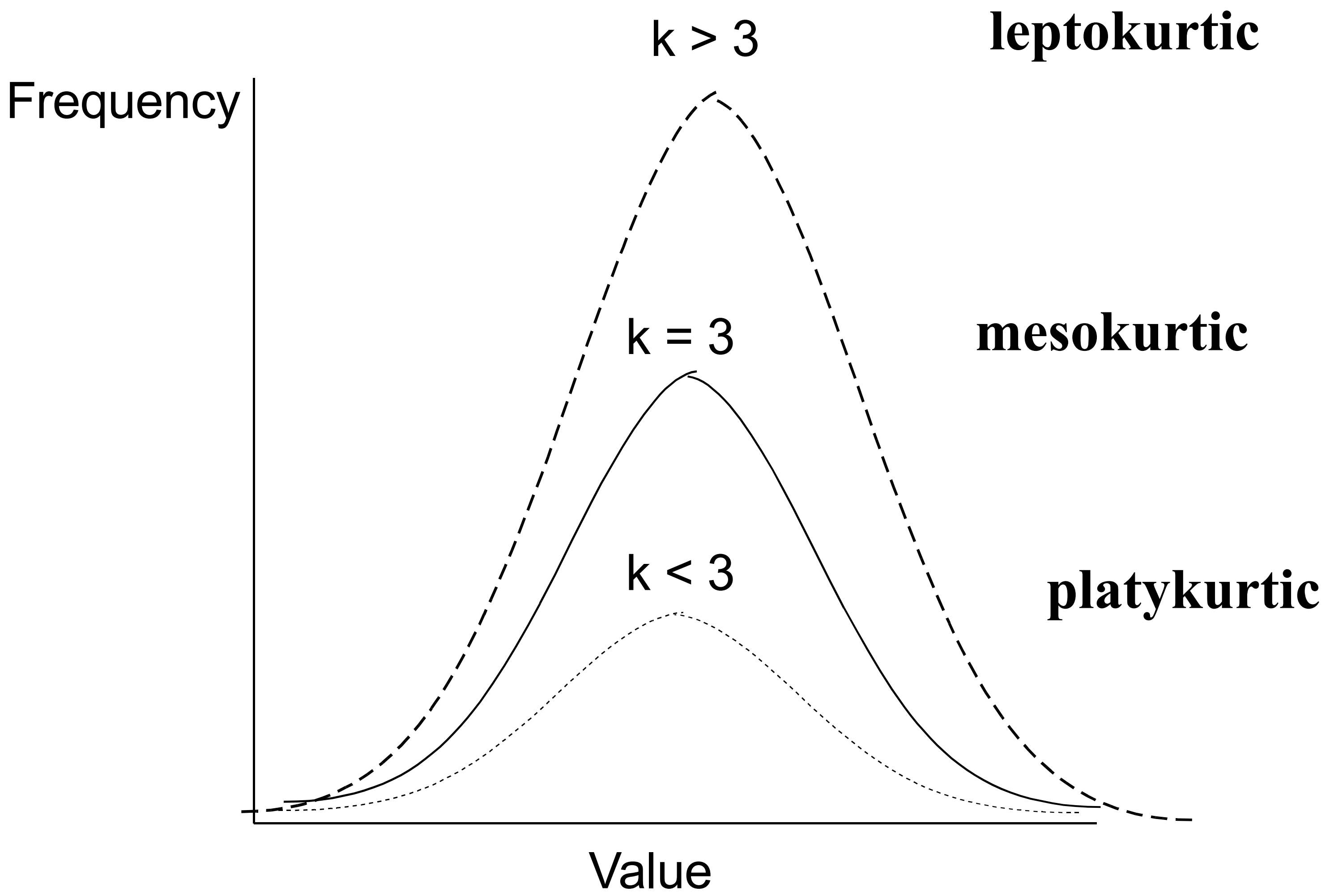
- Income
- Contribution to candidates
- Populations of countries
- “Residual vote” rates

- “Positive skew”
- “Right skew”

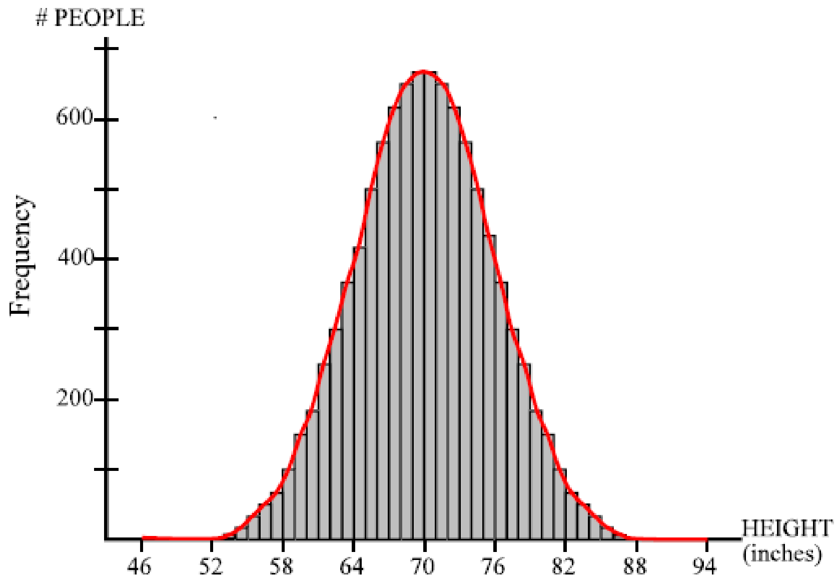
Skewness



Kurtosis


k > 3
leptokurtic
Frequency
k = 3
mesokurtic
k < 3
platykurtic
Value

Normal distribution



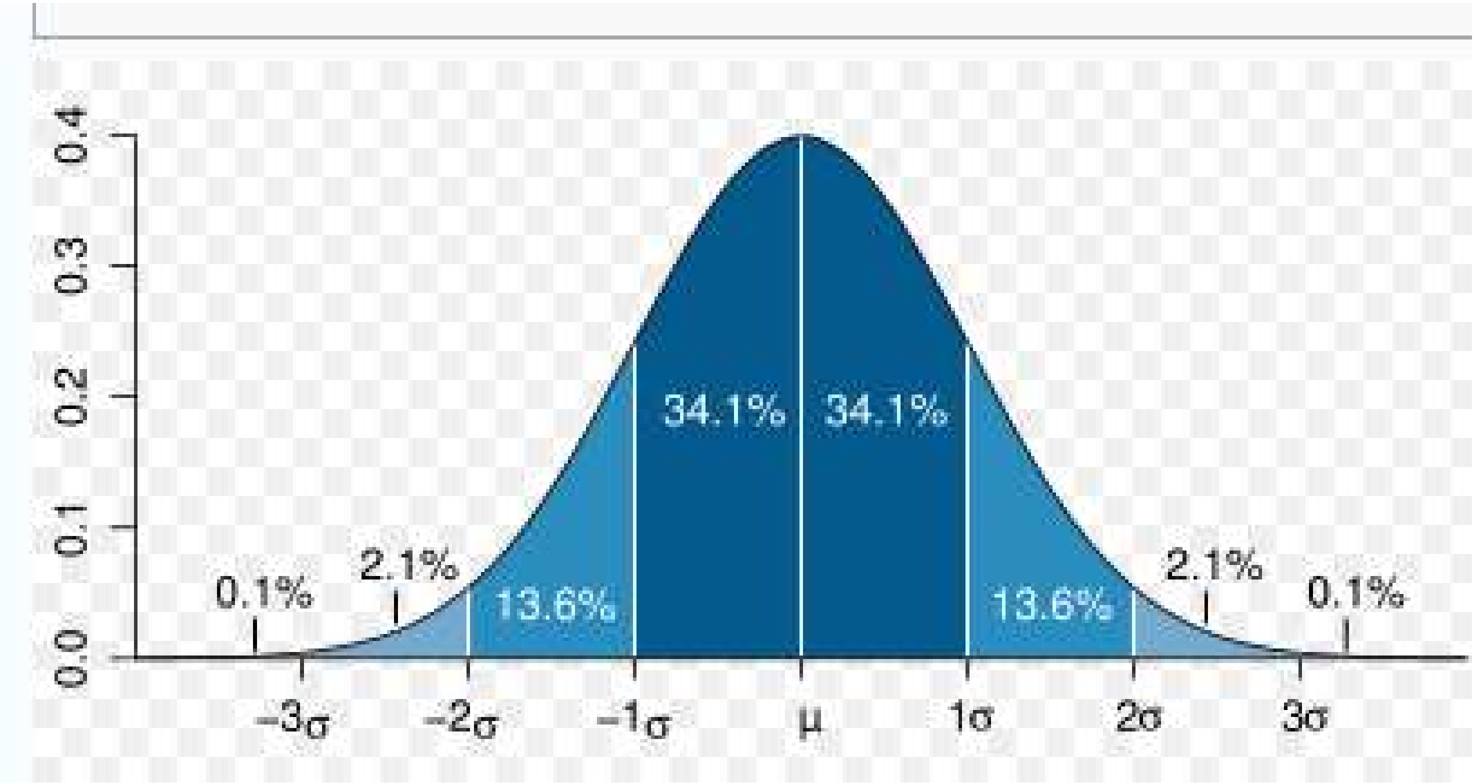
- Skewness = 0
- Kurtosis = 3

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)/2\sigma^2}$$

The *z*-score
or the
"standardized score"

$$z = \frac{x - \bar{x}}{\sigma_x}$$

More words about the normal curve



Commands in STATA for getting univariate statistics

- <u>sum</u>marize *varname*
- <u>sum</u>marize *varname*, <u>det</u>ail
- <u>hist</u>ogram *varname*, *bin() start() width() density/fraction/frequency normal*
- graph box *varnames*
- <u>tab</u>ulate [NB: compare to table]

Example of Sophomore Test Scores

- High School and Beyond, 1980: A Longitudinal Survey of Students in the United States (ICPSR Study 7896)

- *totalscore* = % of questions answered correctly minus penalty for guessing
- *recodedtype* = (1=public school, 2=religious private, 3 = non-sectarian private)

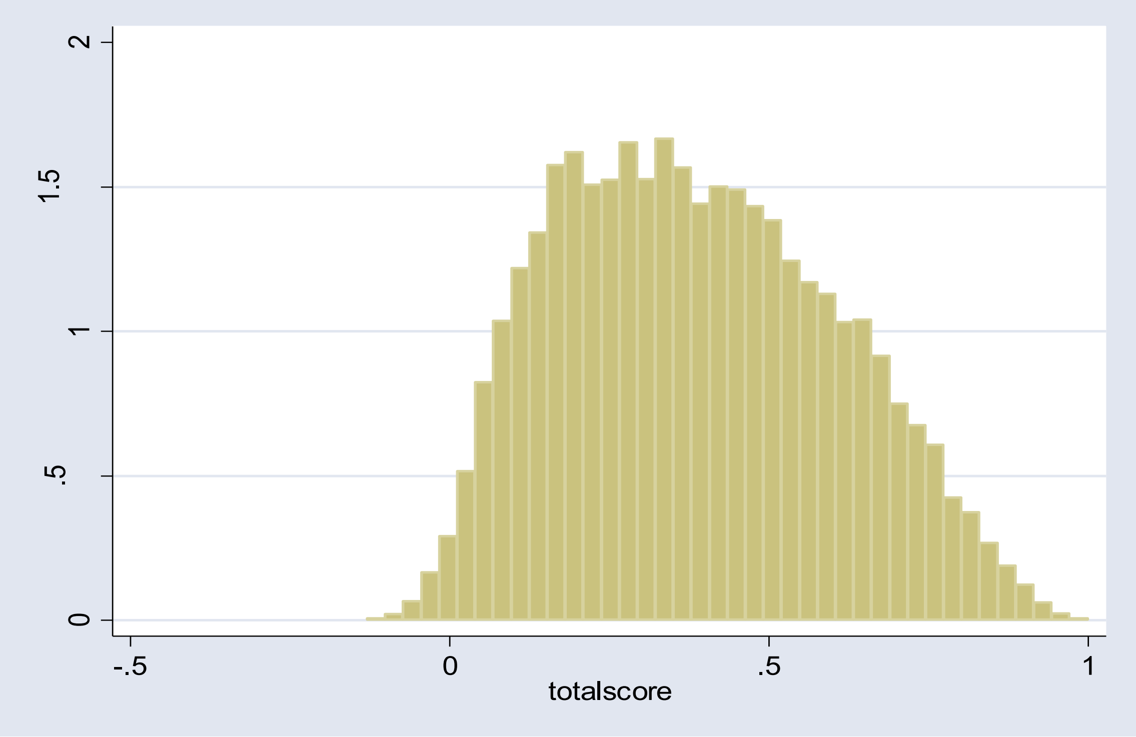
Explore totalscore some more

```
. table recodedtype,c(mean totalscore)

--------------------------
recodedty |
pe        | mean(totals~e)
----------+---------------
        1 |       .3729735
        2 |       .4475548
        3 |        .589883
--------------------------
```

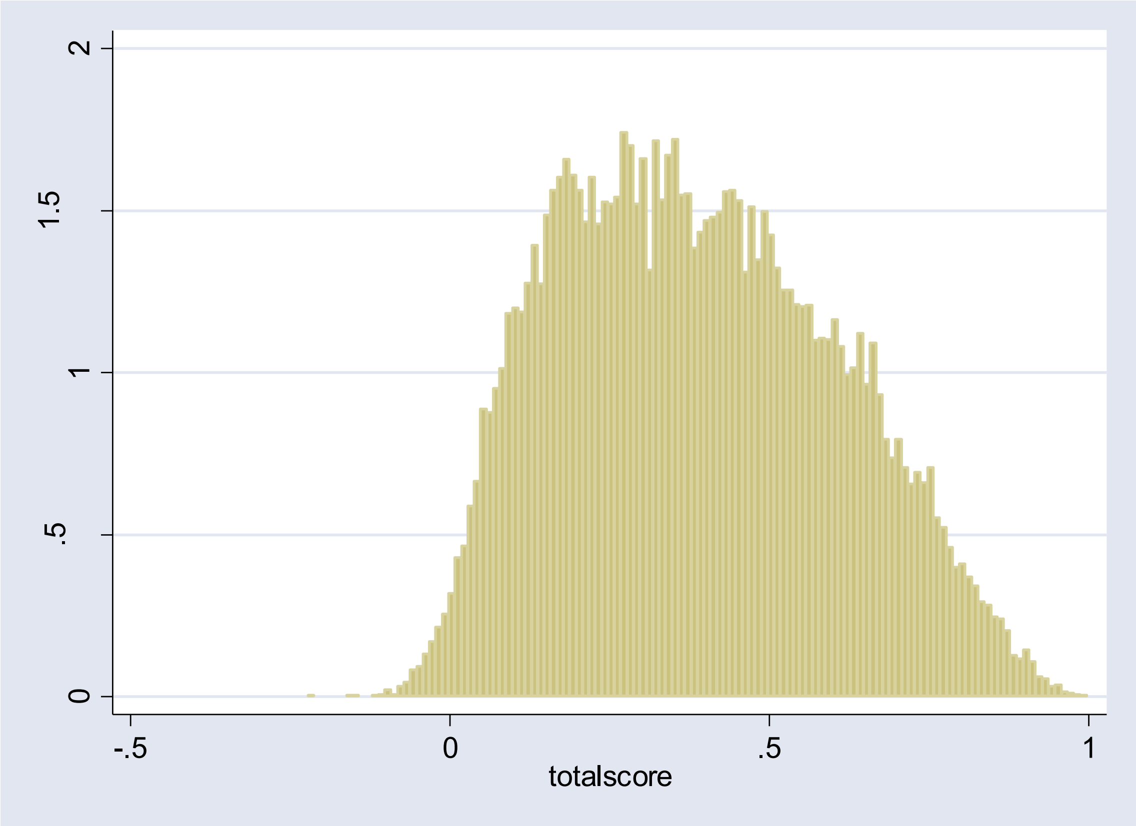
Graph totalscore

```
. hist totalscore
```



Divide into “bins” so that each bar represents 1% correct

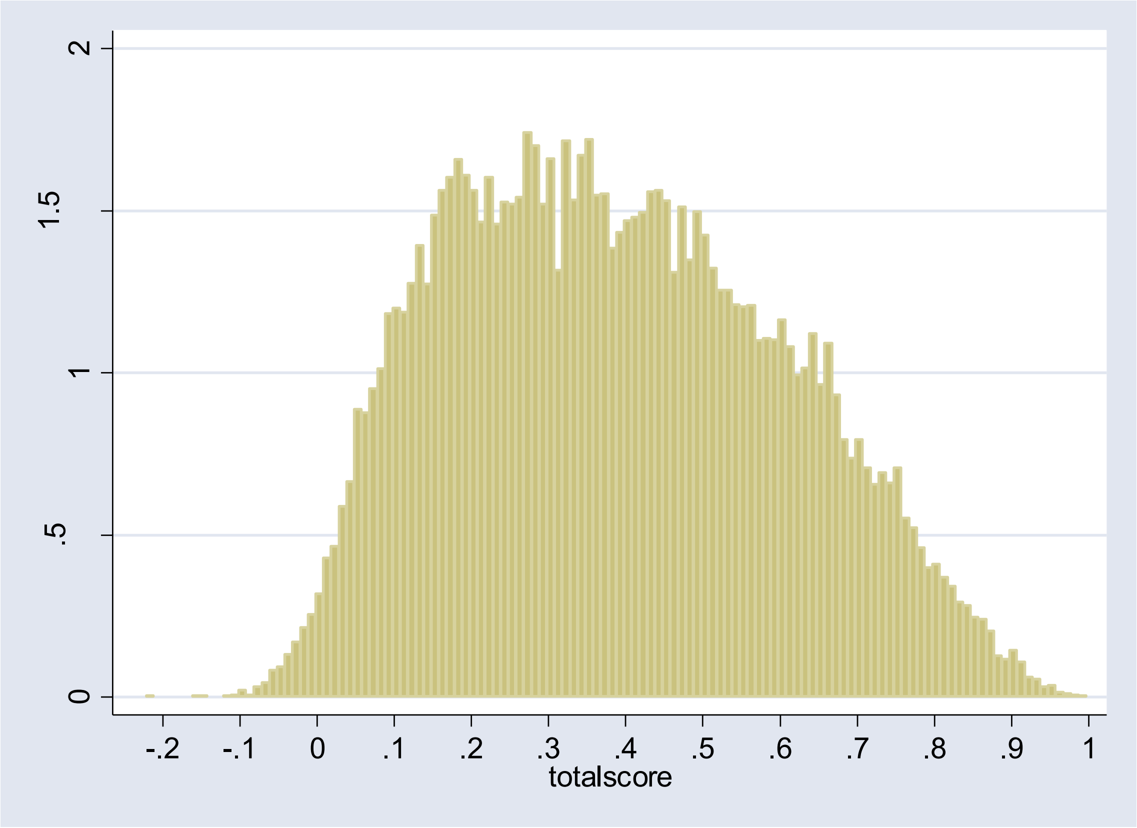
- hist totalscore,width(.01)
- **(bin=124, start=-.24209334, width=.01)**



Add ticks at each 10% mark

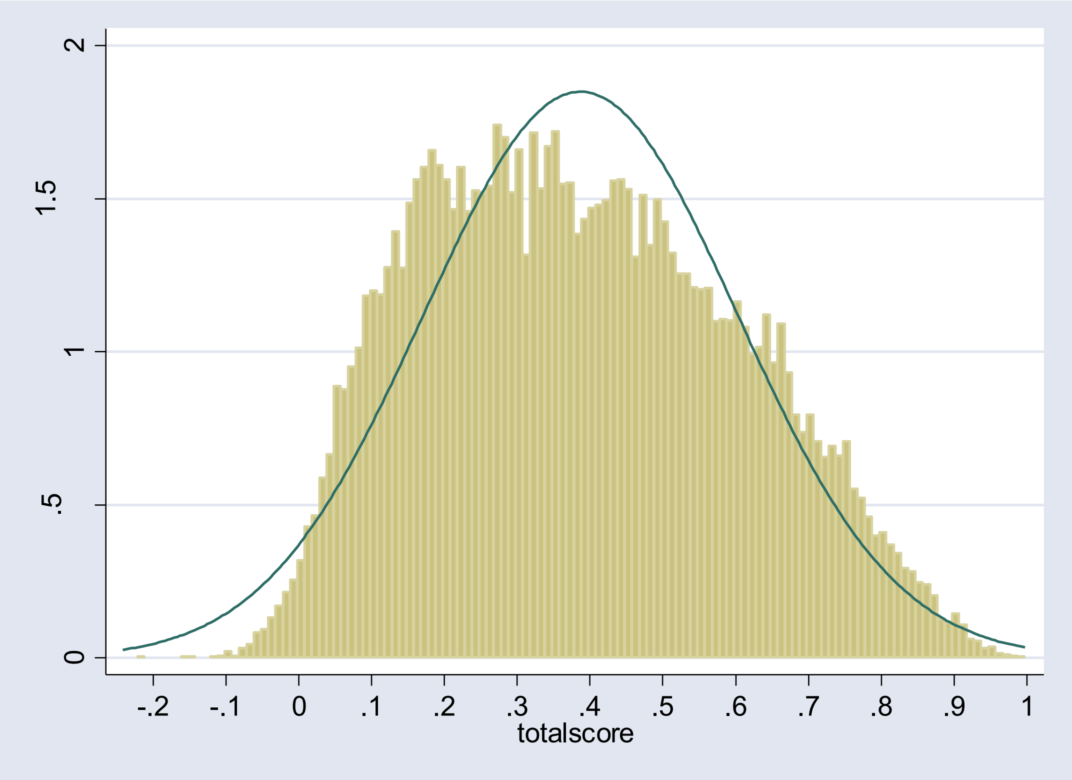
```
histogram totalscore, width(.01) xlabel(-.2 (.1) 1)
```

(bin=124, start=-.24209334, width=.01)



Superimpose the normal curve (with the same mean and s.d. as the empirical distribution)

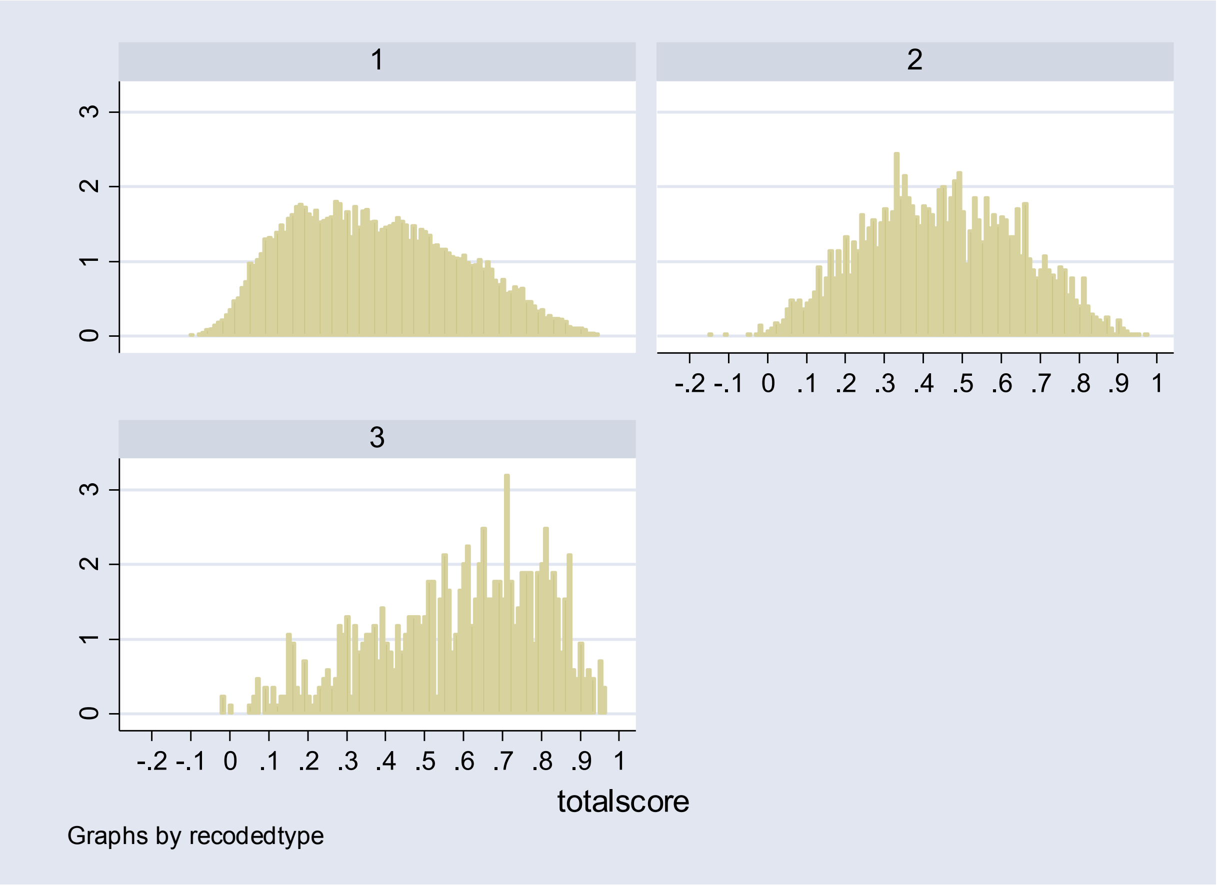
```
. histogram totalscore, width(.01) xlabel(-.2 (.1) 1)
  normal
(bin=124, start=-.24209334, width=.01)
```



Histograms by category

```
.histogram totalscore, width(.01) xlabel(-.2 (.1)1)
  by(recodedtype)
```

(bin=124, start=-.24209334, width=.01)



Main issues with histograms

- Proper level of aggregation
- Non-regular data categories

A note about histograms with unnatural categories

From the Current Population Survey (2000), Voter and Registration Survey

How long (have you/has name) lived at this address?

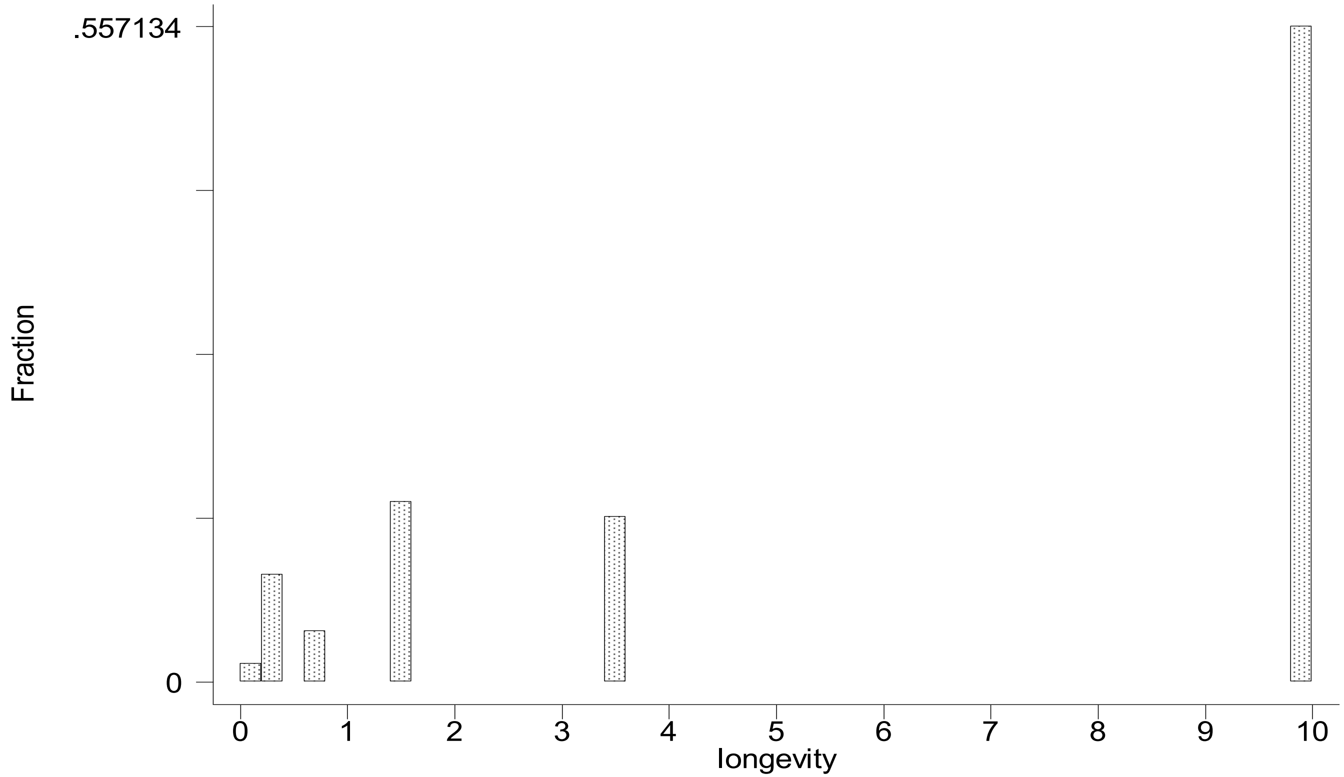
-9 No Response
-3 Refused
-2 Don't know
-1 Not in universe
1 Less than 1 month
2 1-6 months
3 7-11 months
4 1-2 years
5 3-4 years
6 5 years or longer

Solution, Step 1
Map artificial category onto "natural" midpoint

-9 No Response → missing
-3 Refused → missing
-2 Don't know → missing
-1 Not in universe → missing
1 Less than 1 month → 1/24 = 0.042
2 1-6 months → 3.5/12 = 0.29
3 7-11 months → 9/12 = 0.75
4 1-2 years → 1.5
5 3-4 years → 3.5
6 5 years or longer → 10 (arbitrary)

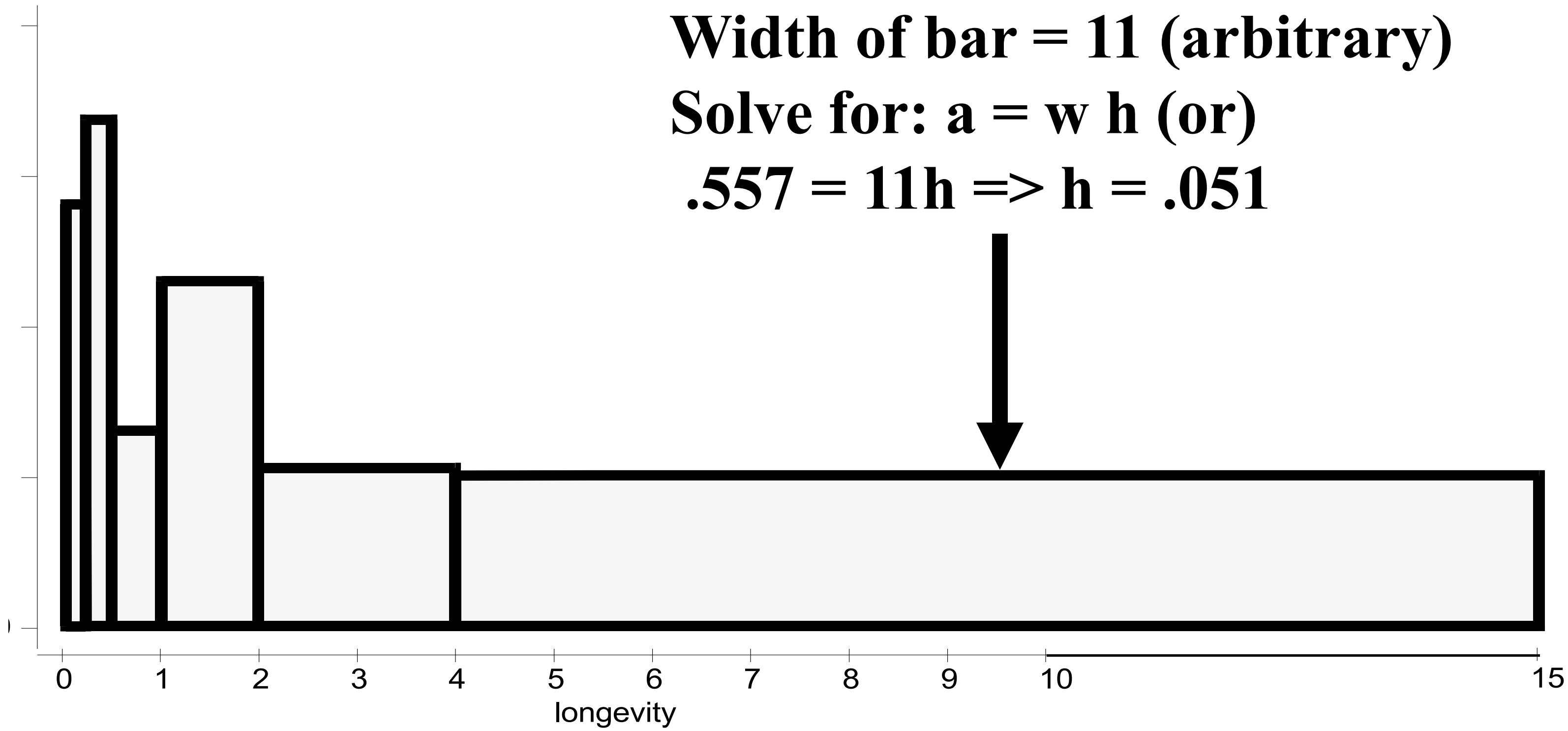
Graph of recoded data

```
histogram longevity, fraction
```



Density plot of data

Total area of last bar = .557
Width of bar = 11 (arbitrary)
Solve for: a = w h (or)
.557 = 11h => h = .051



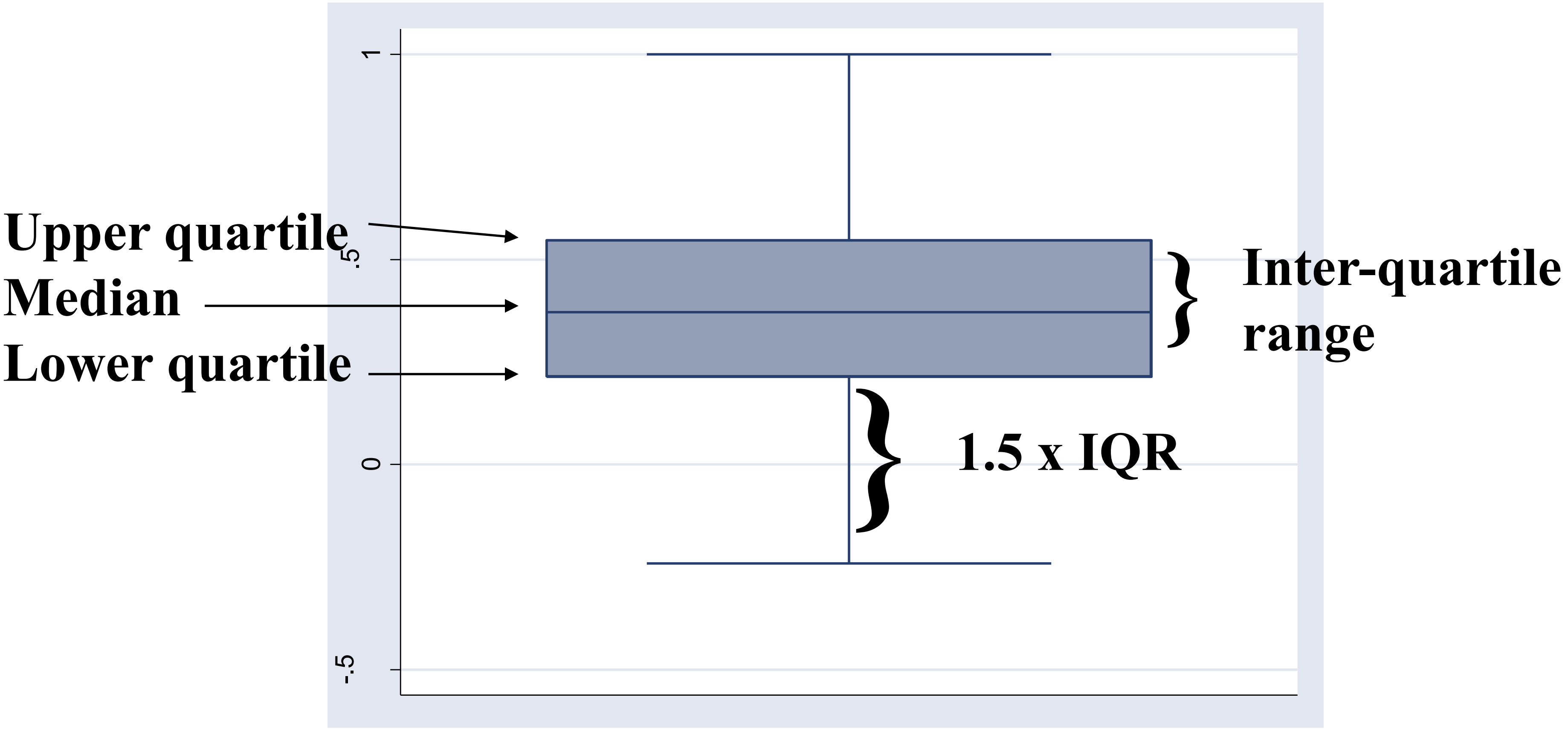
Density plot template

Category	Fraction	X-min	X-max	X-length	Height (density)
< 1 mo.	.0156	0	1/12	.082	.19*
1-6 mo.	.0909	1/12	½	.417	.22
7-11 mo.	.0430	½	1	.500	.09
1-2 yr.	.1529	1	2	1	.15
3-4 yr.	.1404	2	4	2	.07
5+ yr.	.5571	4	15	11	.05

*** = .0156/.082**

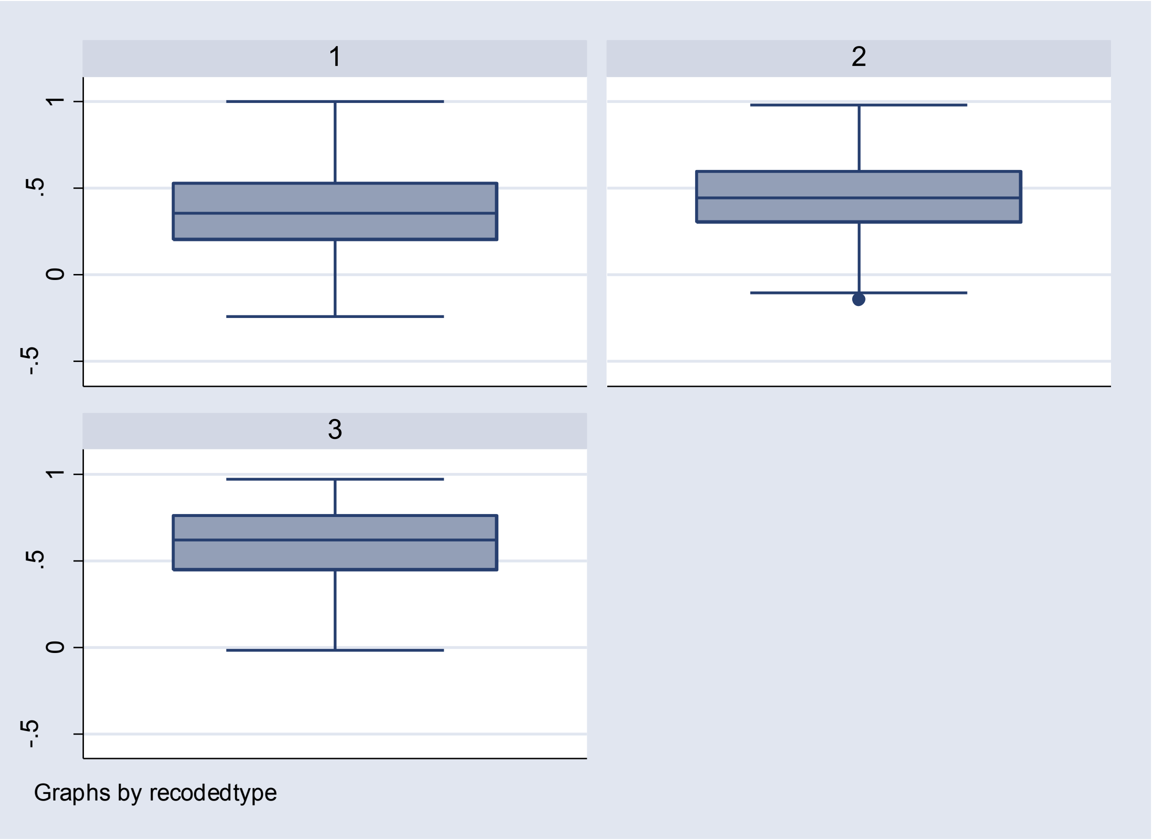
Draw the previous graph with a box plot

```
. graph box totalscore
```



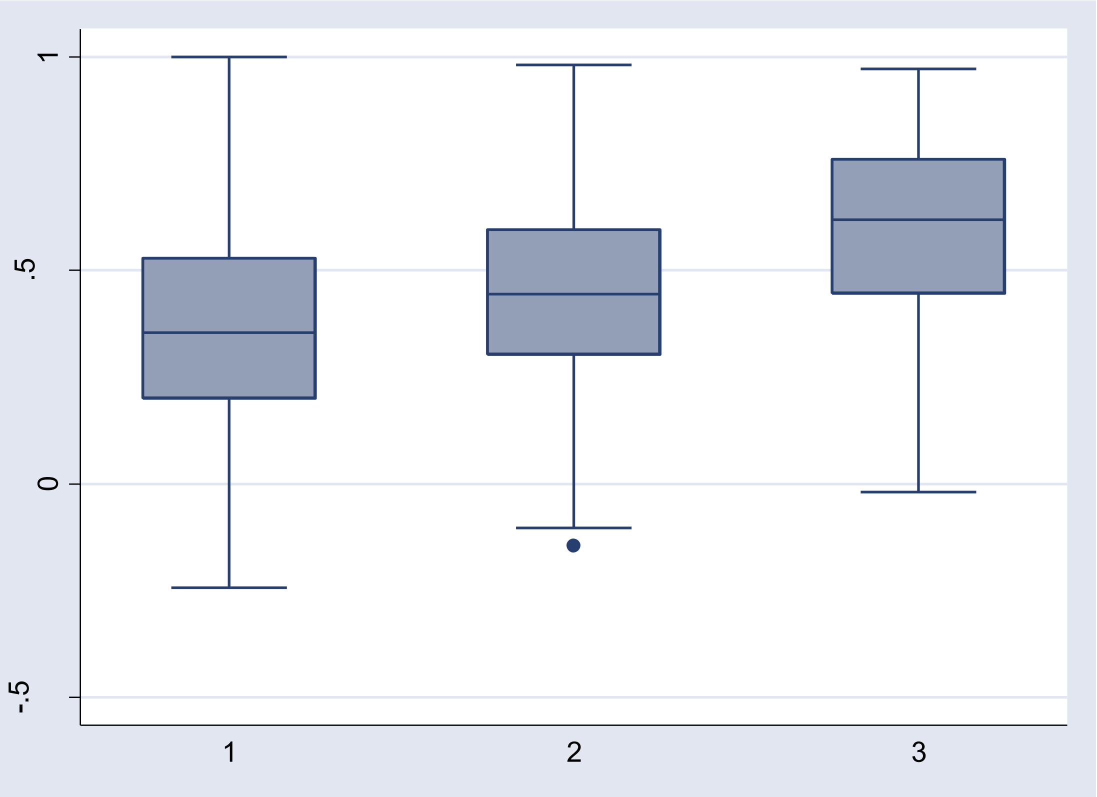
Draw the box plots for the different types of schools

```
. graph box totalscore,by(recodedtype)
```

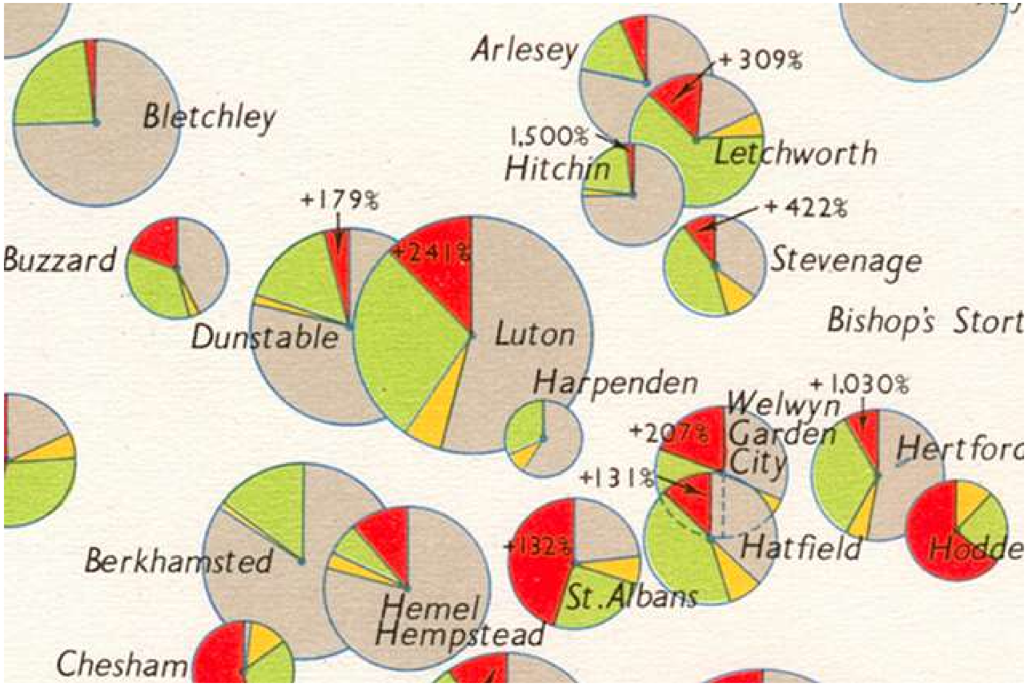


Draw the box plots for the different types of schools using "over" option

```
graph box totalscore,over(recodedtype)
```



Three words about pie charts: don't use them



So, what’s wrong with them

- For non-time series data, hard to get a comparison among groups; the eye is very bad in judging relative size of circle slices
- For time series, data, hard to grasp cross-time comparisons

Some Words about Graphical Presentation

- Aspects of graphical integrity (following Edward Tufte, *Visual Display of Quantitative Information*)
  - Represent number in direct proportion to numerical quantities presented
  - Write clear labels on the graph
  - Show data variation, not design variation
  - Deflate and standardize money in time series